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ON SEQUENCE CONVERGENCE IN $(3, j)$-METRIC SPACES, $j \in \{1, 2\}$

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Abstract. In this article, we show that a convergent sequence in $(3, 2)$-metric spaces has a unique limit. We give several examples in $(3, 1, \rho)$-metric spaces and $(3, 1)$-metric spaces in which a convergent sequence has more than one limit. We obtain sufficient conditions for a sequence in $(3, 1)$-metric space to have a unique limit.

1. Introduction

Many different approaches in generalizing the metric structure and the notions of metric and metric space have appeared in the last few decades. Amongst the generalized metric spaces considered by several authors we mention: Menger [16], Aleksandrov, Nemytskii [1], Mamuzić [15], Gähler [13], Nedev, Choban [20, 22, 21], Kopperman [14], Dhage, Mustafa, Sims [5, 18].

The notion of an (n, m, ρ)-metric, $n > m$, as a generalization of the usual notion of a pseudometric (the case $n = 2, m = 1$), and the notion of an $(n + 1)$-metric (as in [16] and [13]), was introduced in [6]. Connections between some of the topologies induced by a $(3, 1, \rho)$-metric and topologies induced by a pseudo-o-metric, o-metric, and symmetric (as in [22]), is given in [7]. Vast types of characterizations of $(3, j, \rho)$-metrizable topological spaces, $j \in \{1, 2\}$, are given in [3, 4, 9, 10]. The existence and the uniqueness of a fixed point for self-mappings satisfying contractive conditions in $(3, 2)$-W-symmetrizable spaces are proven in [12].

In [3] we define four types of convergence in $(3, j, \rho)$-N-metrizable spaces, $j \in \{1, 2\}$, and give examples that show that in general these four types of convergence are not equivalent in $(3, 1, \rho)$-N-metrizable spaces. On the other hand, in $(3, 2)$-N-metrizable spaces these four types of convergence are equivalent.

In this article, we consider only $(3, j, \rho)$-metric spaces and $(3, j)$-metric spaces, $j \in \{1, 2\}$, in terms of convergence of sequences and examine some properties

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about them. We give several examples in $(3,1,\rho)$-metric spaces and $(3,1)$-metric spaces which show that a convergent sequence in these spaces may have more than one limit. We show that a convergent sequence in $(3,2)$-metric spaces has a unique limit. We give sufficient conditions so the sequence in $(3,1)$-metric space has a unique limit.

2. Preliminaries

We begin this article by establishing notation, basic definitions, and related literature on the subject.
We start with the definitions for $(3,j,\rho)$-metric spaces, $(3,j)$-metric spaces, $j \in \{1,2\}$, as in [3].

Let $M \neq \varnothing$ and $M^{(3)} = M^3/\alpha$, where α is the equivalence relation on M^3 defined by:

$$(x,y,z)\alpha(u,v,w) \Leftrightarrow \pi(u,v,w) = (x,y,z),$$

and π stands for permutation. For simplicity, we use the same notation (x,y,z) for the equivalent class of (x,y,z).
We denote the set of non-negative real numbers with $\mathbb{R}_0^+$.
For a map $d : M^{(3)} \to \mathbb{R}_0^+$, we consider the following conditions:

- $(M0)$ $d(x,x,x) = 0$, for all $x \in M$;
- $(M1)$ $d(x,y,z) \leq d(x,y,a) + d(x,a,z) + d(a,y,z)$, for all $x,y,z,a \in M$;
- $(M2)$ $d(x,y,z) \leq d(x,a,b) + d(y,a,b) + d(z,a,b)$, for all $x,y,z,a,b \in M$;
- (Ms) $d(x,x,y) = d(x,y,y)$, for all $x,y \in M$.

For a subset ρ of $M^{(3)}$, we consider the following conditions:

- $(E0)$ $(x,x,x) \in \rho$, for all $x \in M$;
- $(E1)$ $(x,y,a),(x,a,z),(a,y,z) \in \rho \implies (x,y,z) \in \rho$, for any $x,y,z,a \in M$;
- $(E2)$ $(x,a,b),(y,a,b),(z,a,b) \in \rho \implies (x,y,z) \in \rho$, for any $x,y,z,a,b \in M$.

Definition 2.1. *Let ρ be a subset of $M^{(3)}$.*

- (i) *If ρ satisfies $(E0)$ and (Ej), $j \in \{1,2\}$, we say that ρ is a $(3,j)$-equivalence.*
- (ii) *If ρ satisfies $(E0)$,$(E1)$ and $(E2)$, we say thatρ is a 3-equivalence.*

Example 2.1. It is not difficult to show the following.

- (i) The set $\Delta = \{(x,x,x)|x \in M\}$, is a 3-equivalence on M;
- (ii) The set $\rho = \rho_d = \{(x,y,z)|(x,y,z) \in M^{(3)},\ d(x,y,z) = 0\}$ such that d satisfies $(M0)$ and (Mj), $j \in \{1,2\}$, is a $(3,j)$-equivalence on M.

Definition 2.2. *Let $d : M^{(3)} \to \mathbb{R}_0^+$ and $\rho = \rho_d = \{(x,y,z) \in M^{(3)}|d(x,y,z) = 0\}$.*

- (i) *If d satisfies $(M0)$ and (Mj), $j \in \{1,2\}$, we say that d is a $(3,j,\rho)$-metric on M, and the pair (M,d) is a $(3,j,\rho)$-metric space.*
- (ii) *If d satisfies $(M0)$,(Mj) and (Ms), $j \in \{1,2\}$, we say that d is a $(3,j,\rho)$-symmetric on M, and the pair (M,d) is a $(3,j,\rho)$-symmetric space.*
- (iii) *If d satisfies $(M0)$,$(M1)$ and $(M2)$, we say that d is a $(3,\rho)$-metric on M, and the pair (M,d) is a $(3,\rho)$-metric space.*

(iv) *If d satisfies $(M0)$,$(M1)$,$(M2)$ and (Ms), we say that d is a $(3, \rho)$-symmetric on M, and the pair (M, d) is a $(3, \rho)$-symmetric space.*

If $\rho = \Delta = \{(x, x, x) | x \in M\}$, we write $(3, j)$ instead of $(3, j, \Delta)$.

Example 2.2. Let M be a nonempty set. The map $d : M^{(3)} \to \mathbb{R}_0^+$ defined by:

$$d(x, y, z) = \begin{cases} 0 & , \ x = y = z \\ 1 & , \ \text{otherwise} \end{cases}$$

is a 3-metric on M (the discrete 3-metric).

Proposition 2.1. *If d is a $(3, 2, \rho)$-metric on M, then:*

(i) $d(x, x, y) \leq 2d(x, a, b) + d(y, a, b)$;
(ii) $d(x, x, y) \leq 2d(x, y, y)$;
(iii) $d(x, x, y) \leq 2d(x, z, z) + d(y, z, z)$,

for any $x, y, z, a, b \in M$.

In the rest of the section, we state notations and results from [11].

Definition 2.3. *We say that a sequence (x_n) in a $(3, j, \rho)$-metric space (M, d), $j \in \{1, 2\}$:*

(i) *1-converges to $x \in M$ if $d(x, x, x_n) \to 0$ as $n \to \infty$;*
(ii) *2-converges to $x \in M$ if $d(x, x_n, x_n) \to 0$ as $n \to \infty$;*
(iii) *3-converges to $x \in M$ if $d(x, x_n, x_m) \to 0$ as $n, m \to \infty$.*

We say that the point x is a limit of the sequence (x_n) (with respect to the correspondent convergence).

Theorem 1. *For any sequence (x_n) in a $(3, 2, \rho)$-metric space (M, d), the following conditions are equivalent:*

(i) *(x_n) 1-converges to $x \in M$;*
(ii) *(x_n) 2-converges to $x \in M$;*
(iii) *(x_n) 3-converges to $x \in M$.*

Definition 2.4. *We say that a sequence (x_n) in a $(3, 2, \rho)$-metric space (M, d) is $(3, 2)$-convergent if it satisfies any of the conditions in the previous theorem.*

3. Main results

The main motivation for this article is the convergence-type results, the behavior of sequences, and the relations of different types of convergence, such as Theorem 1, but in the setting of $(3, 1)$-metric spaces. We prove that the limit point of a convergent (in the sense of Theorem 1) sequence is unique in $(3, 2)$-metric spaces and the same statement fails in $(3, 1)$-metric spaces. Moreover, sequences can have infinitely many limits according to the three types of convergence examined in [11].

First, we give an example of a space containing a sequence that 1-converges and 2-converges to any point of that space.

Example 3.1. Let $M = \{0\} \cup \{1/n \,|\, n \in \mathbf{N}\}$ and $d : M^{(3)} \to \mathbb{R}_0^+$ be a map defined by:

$a)$ $d(x,x,x) = 0$, for all $x \in M$;
$b)$ $d(x,x,y) = \min\{x,y\}$, for all $x,y \in M, x \neq y$;
$c)$ $d(x,y,z) = 1$, otherwise.

Then d is a $(3,1,\rho)$-symmetric, but not a $(3,2,\rho)$-symmetric.

Proof. We prove the statement in a few steps. First observe that $d(x,y,z) \leq 1$, for all $x,y,z \in M$.

1^0. Let $\rho = \{(x,y,z) \in M^{(3)} | d(x,y,z) = 0\}$.

2^0. The case $d(x,y,z) \neq 0$.

$2'$. The subcase $d(x,y,z) < 1$. Without loss of generality, we may take $x = z \neq y$.
If $a = x$, then

$$d(x,y,z) = d(a,y,x) \leq d(x,y,a) + d(x,a,z) + d(a,y,z).$$

if $a = y$, then

$$d(x,y,z) = d(x,a,z) \leq d(x,y,a) + d(x,a,z) + d(a,y,z).$$

If $a \notin \{x,y\}$, then

$$d(x,y,z) < 1 = d(x,y,a) \leq d(x,y,a) + d(x,a,z) + d(a,y,z).$$

$2''$. The subcase $d(x,y,z) = 1$. By the definition of d, $z \neq x \neq y \neq z$.
If $a \notin \{x,y,z\}$, then

$$d(x,y,z) = 1 < 3 = d(x,y,a) + d(x,a,z) + d(a,y,z).$$

If $a \in \{x,y,z\}$. Without loss of generality, choose $a = x$. Then

$$d(x,y,z) = d(a,y,z) \leq d(x,y,a) + d(x,a,z) + d(a,y,z).$$

3^0. It is obvious that for all $x,y \in M$,

$$d(x,x,y) = \min\{x,y\} = d(x,y,y)$$

From 1^0, 2^0 and 3^0 it follows that d satisfy the conditions $(M0), (M1)$ and (Ms), i.e. d is a $(3,1,\rho)$-symmetric.

Next, we will prove that d is not a $(3,2,\rho)$-metric.
Let $z \neq x \neq y \neq z$ and $1/3 = x < y < z < 1$. Then by choosing $a = b = 1/3$ one obtains

$$\begin{aligned} d(x,y,z) &= 1 > 0 + 1/3 + 1/3 \\ &= 0 + \min\{x,y\} + \min\{x,z\} \\ &= d(x,a,b) + d(y,a,b) + d(z,a,b). \end{aligned}$$

Thus, d does not satisfy the condition $(M2)$, i.e. d is not a $(3,2,\rho)$-metric.

In the following, we prove that there is a sequence in M, 1-converging to an arbitrary point $x \in M$.

Let $x_n = 1/n \in M$, for $n \in \mathbb{N}$.
1) Let $x = 1/m, m \in \mathbb{N}$, be a fixed point of M.
For a sufficiently large $n \in \mathbb{N}$ we have

$$d(x_n, x, x) = d(1/n, 1/m, 1/m) = \min\{1/n, 1/m\} = 1/n.$$

Thus, $d(x_n, x, x) \to 0$, as $n \to \infty$.
2) Let $x = 0$. Then

$$d(x_n, x, x) = d(1/n, 0, 0) = \min\{1/n, 0\} = 0.$$

Thus, $d(x_n, x, x) \to 0$, as $n \to \infty$.
Together 1) and 2) imply that (x_n) 1-converges to an arbitrary $x \in M$. Since d is a $(3, 1, \rho)$-symmetric, we obtain that (x_n) 1-converges and 2-converges to any point of that space. □

Next, we give an example of a space containing a sequence which 2-converges and 3-converges to any point in that space.

Example 3.2. Let $M = \{1/2^n | n \in \mathbb{N}\}$ and $d : M^{(3)} \to \mathbb{R}_0^+$ be a map defined by:

$$d(x, y, z) = \begin{cases} 0 & , x = y = z \\ \min\{\max\{x, y\}, \max\{y, z\}, \max\{z, x\}\} & , \text{otherwise} \end{cases}.$$

Then d is a $(3, 1)$-metric, but not a $(3, 2)$-metric.

Proof. 1^0. It is obvious that $\rho = \Delta$.
2^0. Let $x, y, z \in M$ and not all of them are equal. Without loss of generality, assume that $x \geq y \geq z$. Then

$$d(x, y, z) = \min\{\max\{x, y\}, \max\{y, z\}, \max\{z, x\}\} = \min\{x, y, x\} = y.$$

Let $a \in M$. We examine the cases:
$2'$. If $a \geq y$, then

$$d(x, y, z) = y = d(a, y, z) \leq d(x, y, a) + d(x, a, z) + d(a, y, z).$$

$2''$. If $a < y$, then

$$d(x, y, z) = y = d(x, y, a) \leq d(x, y, a) + d(x, a, z) + d(a, y, z).$$

From 1^0 and 2^0 it follows that d is a $(3, 1)$-metric.
We will prove that d is not a $(3, 2)$-metric.
For $x = y = 1/2$, $z = 1/4$, $a = b = 1/8$, we obtain

$$d(x, y, z) = 1/2 > 3/8 = d(x, a, b) + d(y, a, b) + d(z, a, b).$$

Thus, d does not satisfy the condition $(M2)$, i.e. d is not a $(3, 2)$-metric.
Our next concern is the convergence. Let $x \in M$. Then

$$\begin{aligned} d(x, 1/2^n, 1/2^n) =& \min\{\max\{x, 1/2^n\}, \max\{1/2^n, 1/2^n\}, \max\{1/2^n, x\}\} \\ & \min\{\max\{x, 1/2^n\}, 1/2^n\} \leq 1/2^n \to 0, \end{aligned}$$

as $n \to \infty$, and

$$d(x, 1/2^n, 1/2^m) = \min\{\max\{x, 1/2^n\}, \max\{1/2^n, 1/2^m\}, \max\{1/2^m, x\}\}$$

$$\leq \max\{1/2^n, 1/2^m\} \to 0,$$

as $n, m \to \infty$.
Since d is a $(3,1)$-metric and x is an arbitrary point of M, we obtained that the sequence (x_n) 2-converges and 3-converges to any point of that space. □

Example 3.3. Let M be a nonempty set, $D : M^2 \to \mathbb{R}_0^+$ be an ordinary metric on M and $k > 0$. Then the map $d : M^{(3)} \to \mathbb{R}_0^+$ defined by:

$$d(x,y,z) = k\Big(D(x,y) + D(y,z) + D(x,z)\Big),$$

is a 3-symmetric and (M,d) is a 3-symmetric space in which every $(3,2)$-convergent sequence has a unique limit (for all three types of convergence given in Theorem 1).

Proof. In [3] it is proven that d is a 3-symmetric. Let (x_n) be a sequence in M and $x, y \in M$ are such that $d(x_n,x,x) \to 0$ and $d(x_n,y,y) \to 0$, as $n \to \infty$. Then $D(x_n,x) \to 0$ and $D(x_n,y) \to 0$, as $n \to \infty$. Since D is an ordinary metric, it follows that $x = y$. □

Theorem 2. *Led $d : M^{(3)} \to \mathbb{R}_0^+$ be a $(3,2)$-metric. Then every (3,2)-convergent sequence has a unique limit (for all three types of convergence given in Theorem 1).*

Proof. Let (x_n) be a (3,2)-convergent sequence in M. Let us suppose that there are $x, y \in M$ such that (x_n) converges to both x and y, i.e. $d(x,x_n,x_n) \to 0$ and $d(y,x_n,x_n) \to 0$, as $n \to \infty$. Then

$$d(x,x,y) \leq 2d(x,x_n,x_n) + d(y,x_n,x_n) \to 0,$$

as $n \to \infty$. From $d(x,x,y) = 0$ and the fact that d is a $(3,2)$-metric, it follows that $x = y$. □

Definition 3.1. *Let $d : M^{(3)} \to \mathbb{R}_0^+$ be a $(3,1,\rho)$-metric on M and the sequence (x_n) i-converges to $x \in M$,$i \in \{1,2,3\}$. Then we say that d is i-sequentially continuous in one variable if for any $a, b \in M$ the sequence $((d(x_n,a,b))$ converges to $d(x,a,b)$.*

Theorem 3. *Let (M,d) be a $(3,1)$-metric space and let d be i-sequentially continuous in one variable, $i \in \{1,2,3\}$. Then every i-convergent sequence has a unique limit in M (with respect to the corresponding convergence).*

Proof. Let (x_n) be an i-convergent sequence in M, $i \in \{1,2,3\}$. Let us suppose that there are $x, y \in M$ such that (x_n) i-converges to both x and y. Since d is i-sequentially continuous in one variable, the sequence $((d(x_n,x,x))$ converges to $d(x,x,x) = 0$ and to $d(y,x,x)$, as well. From the fact that every convergent sequence of reals has a unique limit, it follows that $0 = d(x,x,x) = d(y,x,x)$. So, $x = y$. □

Corollary 3.1. *From the previous examples we can conclude that a $(3,1)$-metric needs not to be i-sequentially continuous in a single variable.*

The last corollary and Examples 3.1 and 3.2 naturally evoke the question of imposing certain conditions on the mapping d to get the uniqueness of the limit of converging sequences. We devote the rest of the article to that question.

Theorem 4. *Let $d : M^{(3)} \to \mathbb{R}_0^+$ be a $(3, 1, \rho)$-metric on M satisfying the condition:*

$$d(u, v, v) \le d(u, w, w) + d(w, v, v),$$

for all $u, v, w \in M$ and let (x_n) be a sequence which 3-converges to $x \in M$.
The sequence (x_n) 1-converges to $x \in M$ iff the sequence of real numbers $(d(x_n, a, a))$ converges to $d(x, a, a)$, for all $a \in M$.

Proof. Let (x_n) be a sequence such that $d(x_n, a, a) \to d(x, a, a)$, as $n \to \infty$, for all $a \in M$. If we take $a = x$, we obtain that $d(x_n, x, x) \to d(x, x, x) = 0$, as $n \to \infty$, i.e. (x_n) 1-converges to x.

Let (x_n) be a sequence which 1-converges to $x \in M$. It is obvious that if a sequence (x_n) 3-converges to x, then also 2-converges to x. Then

$$d(x_n, a, a) \le d(x_n, x, x) + d(x, a, a),$$

i.e.

$$d(x_n, a, a) - d(x, a, a) \le d(x, x, x_n).$$

Similarly one obtains

$$-d(x, x_n, x_n) \le d(x_n, a, a) - d(x, a, a).$$

Thus,

$$-d(x, x_n, x_n) \le d(x_n, a, a) - d(x, a, a) \le d(x, x, x_n).$$

By letting $n \to \infty$ in the last inequality one infers that $(d(x_n, a, a))$ converges to $d(x, a, a)$, for all $a \in M$. □

Corollary 4.1. *Let $d : M^{(3)} \to \mathbb{R}_0^+$ be a $(3, 1)$-metric on M such that:*

$$d(u, v, v) \le d(u, w, w) + d(w, v, v),$$

for all $u, v, w \in M$ and let (x_n) be a sequence which 3-converges to $x \in M$.
The sequence (x_n) has a unique limit $x \in M$ (i.e. (x_n) i-converges to x, for all $1 \le i \le 3$) iff the sequence of reals $(d(x_n, a, a))$ converges to $d(x, a, a)$, for all $a \in M$.

Proof. Suppose $y \in M \setminus \{x\}$ be such that the sequence (x_n) 3-converges and 1-converges to y. Theorem 4 impliy that $d(x_n, y, y) \to d(y, y, y)$ and $d(x_n, y, y) \to d(x, y, y)$ as $n \to \infty$. So, $0 = d(y, y, y) = d(x, y, y)$, implying $x = y$. This is a contradiction. Thus, the limit point is unique. The other direction follows directly from Theorem 4. □

References

[1] P. Alexandroff and V. Niemytzki, *Der allgemeine Metrisationssatz und das Symmetrieaxiom*, Rec. Math. [Mat. Sbornik] N.S. **45**(3) (1938), 663–672.

[2] S. Banach, *Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales*, Fundamenta Mathematicae **3**(1) (1922), 133–181.

[3] S. Čalamani and D. Dimovski, *Topologies induced by* $(3, j, \rho)$*-metrics,* $j \in \{1, 2\}$, International Mathematical Forum **9**(22) (2014), 1075–1088.

[4] S. Čalamani, T. Dimovski and D. Dimovski, *Separation properties for some topologies induced by* $(3, j, \rho)$*-metrics,* $j \in \{1, 2\}$, Math. Nat. Sci. Proceedings of the Sixth International Scientific Conference – FMNS2015 **1** (2015), 24–30.

[5] B. C. Dhage, *Generalized metric spaces mappings with fixed point*, Bull. Calcutta Math. Soc. **84** (1992), 329–336.

[6] D. Dimovski, *Generalized metrics –* (n, m, r)*-metrics*, Mat. Bilten **16**(42) (1992), 73–76.

[7] D. Dimovski, $(3, 1, \rho)$*-metrizable topological spaces*, Math. Maced. **3** (2005), 59–64.

[8] D. Dimovski, $[n, \rho, m]$*-metric spaces*, International conference "Mathematical and Informational Technologies, MIT-2013", Proceedings of papers, Vrnjacka Banja, Serbia, September 5-18, (2013) 111–115.

[9] T. Dimovski and D. Dimovski, *On* $(3, 2, \rho)$*-K-metrizable spaces*, Math. Nat. Sci. Proceedings of the Fifth International Scientific Conference – FMNS2013 **1** (2013), 73–79.

[10] T. Dimovski and D. Dimovski, *Some properties concerning* $(3, 2, \rho)$*-K-metrizable spaces*, Math. Nat. Sci. Proceedings of the Fifth International Scientific Conference – FMNS2013 **1** (2015), 18–23.

[11] T. Dimovski and D. Dimovski, *Convergence of sequences in* $(3, j, \rho)$*-N-metrizable spaces,* $j \in \{1, 2\}$, Mat. Bilten **42**(1) (2018), 21–27.

[12] T. Dimovski and D. Dimovski, *A fixed point theorem for mappings satisfying cyclical contractive conditions in* $(3, 2)$*-W-symmetrizable spaces*, Kragujevac J. Math. **47**(2) (2023), 1021–1028.

[13] S. Gähler, *2-metrische Räume und ihre topologische Struktur*, Math. Nachr. **26** (1963), 115–148.

[14] R. Kopperman, *All topologies come from generalized metrics*, Amer. Math. Monthly **95**(2) (1988), 89–97.

[15] Z. Mamuzič, *Uvod u opštu topologiju I*, Volume 17 of Matematička biblioteka, Zavod za izdavanje udžbenika, Beograd, 1960.

[16] K. Menger, *Untersuchungen über allgemeine Metrik*, Math. Ann. **100** (1928), 75–163.

[17] S. Moradi and A. Beiranvand, *Fixed point of* T_F*-contractive single-valued mappings*, Iranian Journal of Mathematical Sciences and Informatics, **5** (2010), 25–32.

[18] Z. Mustafa and B. Sims, *Some Remarks Concerning D-Metric Spaces*, Proceedings of the International Conferences on Fixed Point Theory and Applications, Valencia (Spain) July (2003), 189–198.

[19] Z. Mustafa and B. Sims, *A new approach to generalized metric spaces*, J. Nonlinear Convex Anal. **7**(2) (2006), 289–297.

[20] S. Nedev, *Generalized-metrizable spaces*, C. R. Acad. Bulgare Sci. **20**(6) (1967), 513–516.

[21] S. Nedev and M. Choban, *On metrization of topological groups*, Vestnik Moskovskogo Universiteta Seriya 1. Matematika Mekhanika. **6** (1968), 18–20.

[22] S. Nedev, *O-metrizable spaces*, Trudy Moskovskogo Matematicheskogo Obshchestva **24** (1971), 201–236.

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