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BEST PROXIMITY POINT RESULTS IN THE ORTHOGONAL METRIC SPACES - BRIEF REVIEW

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Abstract. From the moment when the concept of the orthogonal set appeared in 2017, it become an interesting topic for research. Many authors investigated the uniqueness of best proximal point results in different types of metric space (X, d) where X is an orthogonal set and d is a metric on X. Investigation of those results goes in two main directions. The first type of investigation direction deals with different metric spaces, such as standard metric space, b-metric spaces, metric-like spaces, b metric-like spaces, modular spaces, etc. The second direction deals with different types of contraction. Firstly, we present some necessary definitions with many examples for a better understanding of the concept of the orthogonal set. After that, we give an overview of some results in this field and open problems that could be considered.

1. Introduction

Since it appeared in 1922, the Banach's contraction principle becomes a fundamental tool in pure and applied mathematics. As it is well known, in metric space (X, d) where X is a nonempty set and metric d is continuous, a self mapping T on X with contractive condition $d(Tx, Ty) \leq kd(x, y)$ for $k \in [0, 1)$ has a unique fixed point x, i.e. $Tx = x$.
If we suppose that P and Q are two closed non-empty subsets of X such that $T : P \to Q$ and $P \cap Q = \emptyset$ then the equation $Tx = x$ has no solution. In that case one can find an approximate solution of $Tx = x$ choosing $x \in P$ the closest to $Tx \in Q$. So, if we denote distance between P and Q as $D(P, Q)$ then we can looking for $x \in P$ such that $D(P, Q) = d(x, Tx)$, and such x is a best proximity point of T.
Firstly, best proximity points in the classical metric spaces and generalized metric spaces were investigated (see for example [1], [2], [5], [12] ,[13]).
Recently Gordji et al. [8], [9] suggested the concept of the orthogonal sets with

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some basic terms, and investigated fixed points under weaker, othogonality contractive condition. Since that, many authors investigate best proximal points under orthogonality.
Paper is organized as follows. Firstly, necessary definitions and a few examples are given for better understanding the concept of orthogonality. After that we give a review of some known and our results in this field.

2. Preliminaries

Definition 2.1. [9] Let X be a nonempty set and $\perp \subset X \times X$ be a binary relation. If there exists an element $x_0 \in X$ such that for all $y \in X$ the following hold:

$$y \perp x_0 \quad or \quad x_0 \perp y,$$

then it is called an *orthogonal set* (briefly *O-set*) and x_0 is called an *orthogonal element*. We denote this O-set by $(X, \perp)$.

Example 2.1. [6] Let X be a set of real numbers and define binary relation $\perp$ on X as for all $x, y \in X$ we say that $x \perp y$ if and only if $x + y = x$. Then $x + 0 = x$ is true for every $x \in X$. So, we have that $x \perp 0$ holds for every $x \in X$ and 0 is an orthogonal element.

An interesting real life application of this sophisticated mathematical concept is given in [9].

Example 2.2. [9] Let X be a set of all people in the world. We define $x \perp y$ if x can give blood to y. According to the following table, if x_0 is a person such that his (her) blood type $0-$ then we have $x_0 \perp y$ for all $y \in X$. This means that $(X, \perp)$ is an $O-$set and x_0 is not unique.

Type	You can give blood to	You can receive blood from
A+	A+ AB+	A+ A- 0+ 0-
0+	0+ A+ B+ AB+	0+ 0-
B+	B+ AB+	B+ B- 0+ 0-
AB+	AB+	everyone
A-	A+ A- AB+ AB-	A- 0-
0-	everyone	0-
B-	B+ B- AB+ AB-	B- 0-
AB-	AB+ AB-	AB- B- 0- A-

Note that an orthogonal set does not need to have a unique orthogonal element.

Example 2.3. [6] Let $X = [0, 1]$ and define binary relation $\perp$ on X as for all $x, y \in X$ we say that $x \perp y$ if and only if $x \leq x^y$. Then $x \leq x^0 = 1$ and $x \leq x^1 = x$ is true for every $x \in X$. So, we have that $x \perp 0$ and $x \perp 1$ hold for every $x \in X$, that is 0 and 1 are orthogonal elements. Note that the binary relation in this example is not reflexive since 0^0 is not defined.

Definition 2.2. [9] Let $(X, \perp)$ be an O-set. A sequence $\{x_n\}_{n\in N}$ is called an *orthogonal sequence* (briefly *O-sequence*) if for all $n \in N$ the following holds:

$$x_n \perp x_{n+1} \quad or \quad x_{n+1} \perp x_n.$$

Example 2.4. [6] Let $X = \mathbb{N}$ and let t be an arbitrary element in X. We define binary relation $\perp$ on X as for all $x, y \in X$ we say that $x \perp y$ if and only if $y = x + t$. Then the arithmetic sequence $\{x_n\}$, $x_{n+1} = x_n + t$ is an O-sequence.

Remark 2.1: Note that if we consider an O-sequence $\{x_n\}$ in an O-set $(X, \perp)$, then a subset P consists of the elements of the sequence $\{x_n\}$ is a partially ordered set.

Definition 2.3. [9] Let $(X, \perp)$ be an O-set. Then $f : X \to X$ is said to be *orthogonal preserving* (or *$\perp$-preserving*) if for all $x, y \in X$ such that $x \perp y$ yields $f(x) \perp f(y)$.

Example 2.5. [9] Let $X = [0, 1)$ and define binary relation $\perp$ as $x \perp y$ if and only if $xy \leq \frac{x}{2}$. Then $0 \perp y$ for every $y \in X$. So, 0 is an orthogonal element and $(X, \perp)$ is an O-set.
Let $f : X \to X$ be a mapping defined as $f(x) = \frac{x}{2}$ if $x \leq \frac{1}{2}$ and $f(x) = 0$ if $x > \frac{1}{2}$. Let $x \perp y$ for $x, y \in X$. From $xy \leq \frac{x}{2}$ it follows that $x = 0$ or $y \leq \frac{1}{2}$. So, we have the following cases:

i) $x = 0$ and $y \leq \frac{1}{2}$ implies $f(x) \cdot f(y) = 0 \cdot \frac{y}{2} = 0 = \frac{f(x)}{2}$;
ii) $x = 0$ and $y > \frac{1}{2}$ implies $f(x) \cdot f(y) = 0 \cdot 0 = 0 = \frac{f(x)}{2}$;
iii) $x \leq \frac{1}{2}$ and $y \leq \frac{1}{2}$ implies $f(x) \cdot f(y) = \frac{x}{2} \cdot \frac{y}{2} \leq f(x) \cdot \frac{1}{4} < \frac{f(x)}{2}$;
iv) $x > \frac{1}{2}$ and $y \leq \frac{1}{2}$ implies $f(x) \cdot f(y) = 0 \cdot \frac{y}{2} = 0 = \frac{f(x)}{2}$.

These cases imply that $f(x) \cdot f(y) \leq \frac{f(x)}{2}$, that is f is $\perp$-preserving.

Example 2.6. [6] Let $X = [0, 1] \times [0, 1]$ and for $x = (a_1, b_1)$, $y = (a_2, b_2)$ we define binary relation $\perp$ as $x \perp y$ if and only if $a_1 b_2 = 0$ or $a_2 b_1 = 0$. Then $(0, 0) \perp y$ for every $y \in X$, so $(0, 0)$ is an orthogonal element and $(X, \perp)$ is an O-set. Let $f : X \to X$ be a mapping defined as $f(a, b) = (a^2, b)$. Obviously, we get that if $x \perp y$ then $f(x) \perp f(y)$, i.e. f is $\perp$-preserving.

Definition 2.4. [9] Let $(X, \perp, d)$ be an orthogonal metric space ($(X, \perp)$ is an O-set and (X, d) be a metric space). Then $f : X \to X$ is said to be *orthogonal continuous* (or *$\perp$-continuous*) in $a \in X$ if for each O-sequence $\{a_n\}_{n\in N}$ in X with $a_n \to a$ as $n \to +\infty$, we have $f(a_n) \to f(a)$ as $n \to +\infty$.

Remark 2.2: Every continuous mapping is $\perp$-continuous mapping, but converse is not true. Such an example is given in [9].

Definition 2.5. [9] Let $(X, \perp, d)$ be an orthogonal metric space and $0 \leq \lambda < 1$.
A mapping $f : X \to X$ is called an *orthogonal contraction* (briefly, *$\perp$-contraction*) with Lipshitz constant λ if, for all $x, y \in X$ with $x \perp y$, the following inequality holds

$$d(fx, fy) \leq \lambda d(x, y).$$

Remark 2.3: Every contraction mapping is $\perp$-contraction mapping but converse is not true, for example see [9].

Definition 2.6. [1] Let (X, d) be a metric space and $P, Q \subset X$ such that $P, Q \neq \emptyset$. We define distance between P and Q as $D(P, Q) = \inf\{d(p, q) | p \in P, q \in Q\}$. The point $p \in P$ is a best proximity point (or bpp) of a non-self mapping $T : P \to Q$ if $d(p, Tp) = D(P, Q)$.

Remark 2.4: Distance between two sets $P, Q \subset X$ as introduced in Definition 2.6 is not metric. For example, let (X, d) be a metric space, where $X = [0, 2]$ and $d(x, y) = |x - y|$. Let $P = [\frac{1}{2}, 1]$ and $Q = [\frac{1}{2}, \frac{3}{2}]$. Obviously, $D(P, Q) = 0$, but $P \neq Q$.

For arbitrary sets $P, Q \subset X$ such that $P, Q \neq \emptyset$ we define

$$P_0 = \{p \in P | (\exists q \in Q) d(p, q) = D(P, Q)\},$$

$$Q_0 = \{q \in Q | (\exists p \in P) d(p, q) = D(P, Q)\}.$$

Definition 2.7. [5] Let (X, d) be a metric space and $P, Q \subset X$ such that $P, Q \neq \emptyset$ and $P_0 \neq \emptyset$. Then(P, Q) has P-property if and only if

$$d(p_1, q_1) = D(P, Q) \text{ and } d(p_2, q_2) = D(P, Q) \text{ implies } d(p_1, p_2) = d(q_1, q_2),$$

for all $p_1, p_2 \in P_0$ and $q_1, q_2 \in Q_0$.

Definition 2.8. [12] Let $(X, \preceq)$ be a partially ordered set and (P, Q) be a pair of nonempty subsets of X. A mapping $T : P \to Q$ is called order-preserving if for all $p_1, p_2, q_1, q_2 \in P$ the following holds

$$q_1 \preceq q_2 \text{ and } d(p_1, Tq_1) = d(p_2, Tq_2) = D(P, Q) \text{ implies } p_1 \preceq p_2.$$

Remark 2.5: Regarding the fact that $d(x, Tx) \geq D(P, Q)$ for all $x \in P$, it can be observed that the global minimum of the mapping $x \to d(x, Tx)$ is attained at best proximity point. Moreover, it is easy to see that best proximity point reduces to a fixed point if the underlying mapping T is a self-mapping.

3. Brief literature review - overview of selected results

(1) Gordji et al. in [8] introduced the concept of orthogonal set and proved the next fix point theorem.

Theorem 1. *Let $(X, \perp, d)$ be an O-complete metric space (not necessarily complete metric space) and $0 < \lambda < 1$. Let $f : X \to X$ be $\perp$-continuous, $\perp$-contraction with Lipschitz constant λ and $\perp$-preserving. Then f has a unique fixed point $x' \in X$. Also, f is a Picard operator, that is, $\lim_{n\to\infty} f^n(x) = x'$ for all $x \in X$.*

(2) Gordji and Habibi in [9] proved the next fix point theorem in generalized orthogonal space (the third axiom is $d(x; z) \leq d(x; y) + d(y; z)$ for any points $x; y; z \in X$ such that $x \perp y$, $y \perp z$ and $x \perp z$ considering that if $d(x; y) = +\infty$ or $d(y; z) = +\infty$ then $d(x; y) + d(y; z) = +\infty$).

Theorem 2. *Let $f : X \to X$ be a $\perp$-preserving and $\perp$-continuous map such that* *(a)* $d(fx;fy) \leq \lambda d(x;y)$ *for any points* x *and* y *in*X *such that* $x \perp y$ *and* $0 \leq \lambda < 1$;

(b) *For any point* $x \in X$ *there exists* n_0 *such that for* $(f;\perp)$*-orbit* $\{f^n x\}_{n=0}^{+\infty}$ *we have* $d(f^{n_0}, f^{n_0+1}) < +\infty$;

(c) *If* $x \perp y$, $fx = x$ *and* $fy = y$ *then* $d(x;y) < +\infty$.

Then there exists a unique fixed point x' *of the map* f *and* $lim_{n\to+\infty} f^n x = x'$ *for any point* $x \in X$.

In both above mentioned pepers ([8], [9]) authors applied result to finding the existence of solution for a first-order ordinary differential equation.

(3) Sawangsup et al. in [19] proved the next theorem in standard orthogonal metric space and gave an application to ordinary differential equations.

Theorem 3. *Let* $(X, \perp, d)$ *be an O-complete orthogonal metric space with an orthogonal element* x_0 *and* T *be a self-mapping on* X *satisfying the following conditions:*

(i) T *is* $\perp$*-preserving;*

(ii) T *is* $F_\perp$*-contraction mapping, that is*

$$d(Tx, Ty) > 0 \implies \tau + F(d(Tx, Ty)) \leq F(d(x, y))$$

for all $x, y \in X$, $x \perp y$ *and* $\tau > 0$;

(iii) T *is* $\perp$*-continuous.*

Then, T *has a unique fixed point in* X. *Also, the Picard sequence* $\{T^n(x_0)\}$ *converges to the fixed point of* T.

(4) Fallahi and Eivani in [4] formulate and proved the next best proximity theorem in orthogonal b metric space with assumption of continuity of metric.

Theorem 4. *Assume that* $(X, \perp, d)$ *is an orthogonal complete b-metric space. Also, suppose that* (U, V) *is a pair of non-empty closed subsets of* X *and* $U_0 \neq \emptyset$ *. Moreover, let* $T : U \to V$*be a mapping so that*

(a) T *is a* $\perp$*-preserving and* $\perp$*-proximally increasing mapping provided that* $T(U_0) \subseteq V_0$, *and* (U, V) *has P-property;*

(b) *There exist orthogonal elements* $a_0, a_1 \in U_0$ *provided that*

$$d(a_1, Ta_0) = d(U, V);$$

(c) T *is an O-continuous mapping on* U *provided that*

$$d(Ta, Tb) \leq \lambda Q(a, b)$$

where

$$Q(a, b) = max\{d(a, b), d(a, Ta) - sd(U, V), d(b, Tb) - sd(U, V)\}$$

for all point $a, b \in U$ *so that* $a \perp b$ *and* $\lambda \in [0, \frac{1}{s^2})$.

Then T *has a bpp* $a' \in U$ *so that* $d(a', Ta') = d(U, V)$. *Moreover, if for two bpp(s)* $x, y \in U$ *we have* $x \perp y$, *then* T *has a unique bpp in* U .

Theorem 5. *Assume that* $(X, \perp, d)$ *is an orthogonal complete b-metric space. Also, suppose that* (U, V) *is a pair of non-empty closed subsets of X and* $U_0 \neq \emptyset$ *Moreover, let* $T : U \to V$ *be a mapping so that*

(a) *T is a* $\perp$*-preserving and* $\perp$*-proximally increasing mapping provided that* $T(U_0) \subseteq V_0$*, and* (U, V) *has P-property;*
(b) *There exist orthogonal elements* $a_0, a_1 \in U_0$ *provided that*

$$d(a_1, Ta_0) = d(U, V);$$

(c) *T is an O-continuous mapping on U provided that*

$$d(Ta, Tb) \leq \alpha d(a, b) + \beta d(a, Ta) + \gamma d(b, Tb) - (\beta + \gamma) s d(U, V)$$

where

$$Q(a, b) = max\{d(a, b), d(a, Ta) - sd(U, V), d(b, Tb) - sd(U, V)\}$$

for all point $a, b \in U$ *so that* $a \perp b$ *and* $\alpha, \beta, \gamma \geq 0$ *and* $\alpha s + \beta s + \gamma s^2 < 1$*.*

Then T has a bpp $a' \in U$ *so that* $d(a', Ta') = d(U, V)$*.*

Authors applied those results to get appropriate fix point results.

(5) Gardašević et al. in [6] formulate and prove two theorems for existence and uniqueness of best proximal point of an $\perp$-contraction self mapping in orthogonal $0 - d^{bml}$-complete b-metric-like space without assumption of continuity of metric.

Theorem 6. *Let* $(X, \perp, d^{bml}, s)$ *be an orthogonal* $0 - d^{bml}$*-complete b-metric-like space with* $s \geq 1$*,* (P, Q) *be a pair of two non-empty closed subsets of X having P-property and* $P_0 \neq \emptyset$*. Suppose that a mapping* $T : P \to Q$ *satisfies the following three conditions:*

i) *T is a* $\perp$*-order-preserving and* $T(P_0) \subset Q_0$*;*
ii) *There exist* $a_0, a_1 \in P_0$ *such that* $a_0 \perp a_1$ *and* $d^{bml}(a_1, Ta_0) = D^{bml}(P, Q)$*, where* $D^{bml}(P, Q) = \inf\{d^{bml}(x, y) | x \in P, y \in Q\}$*;*
iii) *T is* $\perp$*-contraction and O-continuous mapping on P with Lipshitz constant* $k \in [0, \frac{1}{s})$*.*

Then T has an unique bpp $a' \in P$*, i.e.* $d^{bml}(a', Ta') = D^{bml}(P, Q)$*.*

Theorem 7. *Let* $(X, \perp, d^{bml}, s)$ *be an orthogonal* $0 - d^{ml}$*-complete b-metric-like space with* $s \geq 1$*,* (P, Q) *be a pair of two non-empty closed subsets of X having P-property and* $P_0 \neq \emptyset$*. Suppose that a mapping* $T : P \to Q$ *satisfied the following three conditions:*

i) *T is a* $\perp$*-order-preserving and* $T(P_0) \subset Q_0$*;*
ii) *There exist* $a_0, a_1 \in P_0$ *such that* $a_0 \perp a_1$ *and* $d^{bml}(a_1, Ta_0) = D^{bml}(P, Q)$*, where* $D^{bml}(P, Q) = \inf\{d^{bml}(x, y) | x \in P, y \in Q\}$*;*
iii) *T is O-continuous mapping on P such that*

$$d^{bml}(Ta, Tb) \leq H(a, b),$$

where,

$$H(a,b) = \alpha_1 d^{bml}(a,b) + \alpha_2 d^{bml}(a,Ta) + \alpha_3 d^{bml}(b,Tb) + \alpha_4 d^{bml}(a,Tb) + \alpha_5 d^{bml}(b,Ta) - C,$$

and $C = \left(\alpha_4 + (\alpha_2+\alpha_3)s + \alpha_5 s^2\right) D^{bml}(P,Q)$, *for all* $a, b \in P$ *such that* $a \perp b$, $\alpha_i > 0, i = 1,2,3,4,5$, $\alpha_2 + s\alpha_5 < \frac{1}{s}$ *and* $s(\alpha_1+\alpha_2) + s^2(\alpha_3 + 2\alpha_5) < 1$. *Then* T *has an unique bpp* $a' \in P$, *i.e.* $d^{bml}(a', Ta') = D^{bml}(P,Q)$.

Authors applied those results to get appropriate fix point results.

Open problems

First, we recall open problem stated in [6]: Whether result presented in Theorem 3.6 holds for $k \in [0,1)$?

In a recent paper [18], the authors have established the existence of the best proximity point and common best proximity point within the O-metric space valid without the transitivity condition. The existence of the best proximity point for a weaker condition called $\perp$- continuity on an O-closed set is proved. Furthermore, authors constructed an example wherein the mapping is not neither continuous nor contraction, yet the best proximity point and common best proximity points can still be determined.

Istratescu type contraction of maps in an orthogonal complete b metric spaces were considered in [7]. As an open research question we single out the generalization of the results obtained therein.

4. Conclusion

The concept of orthogonality is relatively new and poorly researched. So it can be seen as a challenge. Authors believe that further research in this field will bring more connections of best proximity points and orthogonal spaces with different scientific disciplines and that this new concept will have its' place in future research. There are many open topics, whether it's weakening the conditions for the parameters of the existing contract conditions or changing the contract conditions or changing the metric spaces. Consequently, the concept of orthogonality is worth exploring.

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